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A bilinear optimal control problem related to a 3D chemo-repulsion system

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Resumen

We will study a bilinear optimal control problem associated to a 3D chemo-repulsion model with linear production. We prove the existence of weak solutions, and establish a regularity criterion to get global-in-time strong solutions. As a consequence, we deduce the existence of a global optimal solution with bilinear control, and using a Lagrange multiplier theorem in Banach spaces, we derive first-order necessary optimality conditions.

Palabras & frases claves: chemo-repulsion model, strong solutions, bilinear optimal control

1. Introducción

The chemotaxis phenomenon is understood as the directed movement of living organisms in response to chemical gradients. Keller and Segel [1] proposed a mathematical model that describes the chemotactic aggregation of cellular slime molds. These molds move preferentially towards relatively high concentrations of a chemical substance secreted by the amoebae themselves. Such mechanism is called *chemo-attraction* with production. Conversely, when the regions of high chemical concentration generate a repulsive effect on the organisms, the phenomenon is called *chemo-repulsion*.

Bilinear control problems are a special class of nonlinear control problems, in which a nonlinear term is constructed by multiplication of the control and state

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variables. In fact, the control acts as the coefficient of a reaction term depending linearly on the state. In this work we study an optimal control problem subject to a chemo-repulsion system with linear production term, and in which a bilinear control acts injecting or extracting chemical substance on a subdomain of control $\Omega_c \subset \Omega$. Specifically, let $\Omega \subset \mathbb{R}^3$ be a bounded domain with boundary $\partial\Omega$ of class $C^{2,1}$ and $(0,T)$ a time interval, with $0 < T < +\infty$. Then, a control problem is studied, which is related to the following system in the time-space domain $Q := (0,T) \times \Omega$:

$$\left\{ \begin{array}{rcl} \partial_t u - \Delta u & = & \nabla \cdot (u \nabla v), \\ \partial_t v - \Delta v + v & = & u + f\, v\, 1_{\Omega_c}, \end{array} \right. \tag{1}$$

with initial conditions

$$u(0,\cdot) = u_0 \geq 0,\ v(0,\cdot) = v_0 \geq 0 \ \text{ in } \Omega, \tag{2}$$

and non-flux boundary conditions

$$\frac{\partial u}{\partial \mathbf{n}} = 0, \quad \frac{\partial v}{\partial \mathbf{n}} = 0 \ \text{ on } (0,T) \times \partial\Omega, \tag{3}$$

where $\mathbf{n}$ denotes the outward unit normal vector to $\partial\Omega$. In (1), the unknowns are the cell density $u(t,x) \geq 0$ and chemical concentration $v(t,x) \geq 0$. The function $f = f(t,x)$ denotes a bilinear control acting in the chemical equation. It is observed that, in the subdomains of Ω_c where $f \geq 0$, such a control acts as a proliferation coefficient of the chemical substance, and conversely where $f \leq 0$, the control acts as a degradation coefficient of the chemical substance. In particular, with this choice of the bilinear control, the solution (u,v) of system (1)-(3) always remains nonnegative. By considering a distributed control with a negative sign, the positivity of v could not be guaranteed.

Referencias

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