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Besov Maximal regularity for a class of degenerate integro-differential equations with infinite delay in Banach spaces.

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Autores. Rafael Aparicio [*] and Valentin Keyantuo [**]

Resumen

The theory of operator-valued Fourier multipliers is used to obtain characterizations for well-posedness of a large class of degenerate integro-differential equations of second order in time in Banach spaces. Specifically, we treat the case of vector-valued Besov spaces on the real line. It is important to note that in particular, the results are applicable to the more familiar scale of vector-valued Hölder spaces. The equations under consideration are important in several applied problems in physics and material science, in particular for phenomena where memory effects are important. Several models in the area of viscoelasticity, including heat conduction and wave propagation correspond to the general class of integro-differential equations considered here. The importance of the results is that they can be used to treat nonlinear equations.

Palabras & frases claves: Well-posedness, maximal regularity, operator-valued Fourier multiplier, Besov spaces, Hölder spaces, material with memory .

1. Introducción

We establish well-posedness results for the following general problem which consists in a degenerate second order non homogeneous integro-differential equa-

[*]Universidad de Puerto Rico, e-mail: rafael.aparicio@upr.edu
[**]Universidad de Puerto Rico, e-mail: valentin.keyantuo1@upr.edu

tion with infinite delay in a Banach space:

$$\begin{aligned}(Mu')'(t) - \Lambda u'(t) - \frac{d}{dt}\int_{-\infty}^{t} c(t-s)u(s)ds \\ = \gamma u(t) + Au(t) + \textstyle\int_{-\infty}^{t} b(t-s)Bu(s)ds + f(t), \text{ for a.e. } t \in \mathbb{R}.\end{aligned} \tag{1}$$

Here, A, B, Λ and M are closed linear operators in a Banach space X satisfying the assumption $D(A) \cap D(B) \subset D(\Lambda) \cap D(M)$, $b, c \in L^1(\mathbb{R}_+)$, f is an X-valued function defined on $\mathbb{R}$, and γ is a constant. If $M \equiv 0$, the above reduce to degenerate first order integro-differential equation. Equations of first and second order in time are of interest. Equations of the form (1) appear in a variety of applied problems such as viscoelasticity and thermoelasticity, when memory effects are present.

Our main tool is the theory of operator-valued Fourier multipliers in the vector-valued Besov spaces $B^s_{pq}(\mathbb{R}, X)$, $s > 0$, $1 \le p, q \le \infty$. Operator-valued Fourier multipliers have been used to study maximal regularity in Besov spaces of evolutionary differential equations in Banach spaces was done starting with [1, 2] by H. Amman and [5] by Girardi-Weis for the evolutionary problem and by Arendt-Bu [4] for periodic boundary conditions. In [3] by Arendt-Batty-Bu was used operator-valued Fourier multipliers to study maximal regularity of evolutionary differential equations in Banach spaces in the scale of vector valued Hölder spaces in both evolutionary problem and periodic boundary conditions. The theory of operator-valued Fourier multipliers allows us to work without the restriction to Hilbert space imposed by the use of Plancherel's theorem. Also, we do not need to assume that the operators involved are semigroup or cosine function generators.

The well-posedness or maximal regularity results are important in that they allow for the treatment of nonlinear problems. In fact both semilinear and quasilinear equations can be handled with this method.

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