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Adaptive significance levels in linear regression models with unknown-variance

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Alejandra Estefanía Patiño Hoyos*, Victor Fossaluza †

Abstract

Keywords: Adaptive significance levels, Bayesian test, e-value, Linear regression, Predictive distribution, Significance test. .

The Full Bayesian Significance Test (FBST) for precise hypotheses is presented by Pereira and Stern (1999) as a Bayesian alternative to the traditional significance tests based on p-values. With the FBST the authors introduce the e-value as an evidence index in favor of the null hypothesis (**H**). An important practical issue for the implementation of the FBST is to establish how small the evidence against **H** must be in order to decide for its rejection. In the FBST procedure, the e-value exhibits similar behavior to the p-value when the sample size increases, i.e., tends to decrease, which suggests that the cut-off point to define the rejection of **H** should be a sample size function. In this work we present a method to find a cutoff value for the evidence in the FBST by minimizing the linear combination of the averaged type-I and type-II error probabilities for a given sample size and also for a given dimensionality of the parameter space. For that purpose, the scenario of linear regression models with unknown-variance under the Bayesian approach is considered.

*e-mail: alejaeph@ime.usp.br
†IME, University of São Paulo, Brazil, e-mail: victorf@ime.usp.br

1 Introduction

The Full Bayesian Significance Test (FBST) for precise hypotheses is presented by [1] as a Bayesian alternative to the traditional significance tests based on p-values. With the FBST the authors introduce the e-value as an evidence index in favor of the null hypothesis (**H**). An important practical issue for the implementation of the FBST is to establish how small the evidence must be to decide to reject **H**. In that sense, [2] present loss functions such that the minimization of their posterior expected value gives "Bayesianity" to the FBST, having a characterization within the Decision Theory approach. This procedure provides a cutoff point for the evidence that depends on the severity of the error for deciding whether to reject or accept **H**.

In the frequentist significance-test context, it is known that the p-value decreases as sample size increases, so by setting a single significance level, it usually leads to rejection of the null hypothesis. In the FBST procedure, the e-value exhibits similar behavior to the p-value when the sample size increases, which suggests that the cutoff point to define the rejection of **H** should be a function of sample size. However, in the proposal of [2], no loss functions that explicitly take into account the sample size are studied.

In order to solve the problem of testing hypotheses in the usual way, in which changing the sample size influences the probability of rejecting or accepting the null hypothesis, [3] motivated by [4], suggests that the level of significance in hypothesis testing should be a function of sample size. Instead of setting a single level of significance, [3] proposes fixing the ratio of severity between type-I and type-II error probabilities based on the incurred losses in each case, and thus, given a sample size, defining the level of significance that minimizes the linear combination of the decision error probabilities. [3] shows that, by increasing the sample size, the probabilities of both kind of errors and their linear combination decrease, when in most cases, setting a single level of significance independent of sample size, only type-II error probability decreases. The tests proposed by [3] takes the same conceptual grounds of the usual tests for simple hypotheses based on the Neyman-Pearson Lemma as presented in [5]. [3] extends the idea to composite and sharp hypotheses, according to the initial work of [4].

Linear models are probably the most used statistical models to establish the influence of a set of covariates on a response variable. In that sense, the proper identification of the relevant variables in the model is an important issue in any scientific investigation, being a more challenging task in the context of Big-Data problems. In addition to high dimensionality, in recent statistical learning problems, it is common to find large datasets with thousands of observations. This fact may cause the hypothesis of nullity of the regression coefficients to be rejected, most of the time, due to the large sample size when the significance level is fixed.

The main goal of our work is to determine how small the Bayesian evidence in the FBST should be in order to reject the null hypothesis. Therefore, taking into account the concepts in [5] and [4] associated with optimal hypothesis tests, as well as the conclusions of [3] about the relationship between the significance levels and the sample size, and finally, considering the ideas developed recently by [6] and [7] related to adaptive significance levels, we present a method to find a cutoff point for the e-value by minimizing a linear combination of the averaged type-I and type-II error probabilities for a given sample size and also for a given dimensionality of the parameter space. For that purpose, the scenario of linear regression models with unknown-variance under the Bayesian approach is considered. So, by providing an adaptive level for decision making and controlling the probabilities of both kind of errors, we intend to avoid the problems associated with the rejection of the hypotheses on the regression coefficients when the sample size is very large.

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