II Encuentro Matemático del Caribe

Cartagena de Indias - Colombia

Septiembre 9 - 12, 2020

Expansiveness, Shadowing and Markov Partition for Anosov Families

Authors: Jeovanny Muentes Acevedo, Raquel Ribeiro

Universidad Tecnológica de Bolívar

E-mail: **jmuentes@utb.edu.co**

Abstract: We study Anosov families which are sequences of diffeomorphisms along compact Riemannian manifolds such that the tangent bundle split into expanding and contracting subspaces. In this paper we prove that a certain class of Anosov families: admit canonical coordinates, are expansive, satisfy the shadowing property, and exhibit a Markov partition.

Keywords: Anosov families, Anosov diffeomorphisms, Markov partitions, uniform hyperbolicity, non-autonomous dynamical systems.

Introduction

An *Anosov family* is a (biinfinite) sequence of diffeomorphisms along a sequence of compact Riemannian manifolds, with an invariant sequence of splittings of the tangent bundle into expanding and contracting subspaces, and with a uniform upper bound for the contraction and lower bound for the expansion.

Anosov families were introduced by P. Arnoux and A. Fisher in [11], motivated by generalizing the notion of Anosov diffeomorphisms. The authors concentrated their studies on linear Anosov families on the two-torus. The first goal was to get a natural notion of completion for the colletion of the set of all orientation-preserving linear Anosov diffeomorphisms on the two-torus (see [10, 11]). Authors have been study Anosov families. Young [15] proved that families consisting of C^{1+1} perturbations of an Anosov diffeomorphism of class C^2 are Anosov families. Recently, Muentes, studied in his doctoral thesis, [2, 3, 4, 5], the Stable and Unstable Manifold Theorem for Anosov family and the stability structural of Anosov families on compact Riemannian manifolds. Ribeiro et al. in [6, 7, 8, 9] studied severals types of shadowing.

In [1] we will study some properties related to hyperbolicity in the Anosov families: *expansiveness, Shadowing and Markov Partition* (see [12, 13, 14]).

References

[1] Muentes, J., & Ribeiro, R. (2020). Expansiveness, Shadowing and Markov Partition for Anosov Families. *arXiv preprint arXiv:2007.07424.*

[2] Acevedo, J. D. J. M. *Famílias Anosov: estabilidade estrutural, variedades invariantes, e entropía para sistemas dinâmicos não-estacionários* (Doctoral dissertation, Universidade de São Paulo).

[3] Acevedo, J. D. J. M. (2018). Openness of Anosov families. *Journal of the Korean Mathematical Society, 55*(3), 575-591.

[4] Acevedo, J. D. J. M. (2019). Local stable and unstable manifolds for Anosov families. *Hokkaido Mathematical Journal, 48*(3), 513-535.

[5] Muentes Acevedo, J. D. J. (2019). Structural stability and a characterization of Anosov families. *Dynamical Systems, 34*(3), 399-421.

[6] Ribeiro, R. (2014). Hyperbolicity and types of shadowing for C^1 generic vector fields. *Discrete & Continuous Dynamical Systems-A, 34*(7), 2963.

[7] Arbieto, A., Reis, J. E., & Ribeiro, R. (2015). On various types of shadowing for geometric Lorenz flows. *Rocky Mountain Journal of Mathematics, 45*(4), 1067-1090.

[8] Arbieto, A., & Ribeiro, R. (2011). Flows with the (asymptotic) average shadowing property on three-dimensional closed manifolds. *Dynamical Systems, 26*(4), 425-432.

[9] Bessa, M., & Ribeiro, R. (2015). Conservative flows with various types of shadowing. *Chaos, Solitons & Fractals, 75, 243-252*.

[10] Arnoux, P., & Fisher, A. M. (2002). Anosov families, random dynamics and the scenery flow. *preprint*.

[11] Arnoux, P., & Fisher, A. M. (2005). Anosov families, renormalization and nonstationary subshifts. *Ergodic Theory and Dynamical Systems, 25*(3), 661-710.

[12] Bowen, R. *Markov partitions for Axiom A diffeomorphisms*, Trans. Amer. Math. Sot. 154 (1971) 377- 397.

[13] Bowen, R. *Equilibrium States and the Ergodic Theory of Anosov Diffeomorphisms*, Lecture Notes in Mathematics, Vol. 470 (Springer-Verlag, New York, 1975).

[14] Fathi, A. *Expansiveness, hyperbolicity and Hausdorff dimension.* Comm. Math. Phys. 126 (1989), no. 2, 249 - 262.

[15] Young L. S., *Stochastic stability of hyperbolic attractors*, Ergodic Theory Dynam. Systems 6 (1986), no. 311-319.